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UH-6149

B. E. - II (Sem. III) (Civil) Examination

May / June – 2012

Engg. Mathematics & Statistical Methods

Time : **3** Hours] [Total Marks : **100**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ B. E. - 2 (SEM. 3) (CIVIL)

Name of the Subject :

☛ ENGG. MATHEMATICS & STATISTICAL METHODS

☛ Subject Code No. : 6 1 4 9 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures on right indicate full marks.

(4) Assume suitable data wherever necessary.

1 (a) Attempt the following : **10**

(1) Prove that $\overline{F} = (2x + yz)i + (4y + zx)j - (6z - xy)k$ is solenoidal.

(2) State Legendre's Duplication formula.

(3) Change the order of integration $\int_0^1 \int_{4y}^{y} e^{x^2}\, dA$.

(4) Evaluate $\int_1^2 \int_0^x \frac{dA}{x^2 + y^2}$

(5) State Stoke's theorem.

(b) Attempt the following : **10**

(1) State and prove the relation between Beta-Gamma function. **4**

(2) Prove that $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx = \frac{n!}{(m+1)n+1}$. **3**

(3) Evaluate $\int_0^{\pi/2} \sin^3 x \cos^{5/2} x dx$. **3**

2 Attempt any **three** : **12**

(1) Evaluate $\iint_R \left(x^2 + y^2\right) dA$ through the area enclosed by the curve $y = 4x$, $x + y = 3$, $y = 0$, $y = 2$.

(2) Find volume bounded by the *xy*-plane, paraboloid $2z = x^2 + y^2$ and cylinder $x^2 + y^2 = 4$.

(3) Solve by changing order of integration

$$\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xydxdy .$$

(4) Find area enclosed within the curve $y = 2 - x$, $y^2 = 2(2 - x)$.

3 (a) Attempt any **two** : **8**

(1) Find directional derivative of $f(x, y, z) = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of outer normal $i + 2j + 2k$.

(2) Show that

$$\overline{F} = \left(y^2 \cos x + z^3\right) i + (2y \sin x - 4) j_2 + \left(3xz^2 + 2\right) k$$

is irrotational. Find its scalar potential function also.

(3) If $\overline{F} = 2xyzi + \left(x^2 y\right) k + \left(x^2 z + 2y\right) j$ then if $\overline{F}$ is conservative (i) find its scalar potential ϕ. (ii) Find the work done in moving a particle under this force field from $(0, 1, 1)$ to $(1, 2, 0)$.

(b) Attempt any **two** : **10**

(1) Evaluate $\oint \overline{F} \cdot dr$ where $\overline{F} = y^2 i + xyj + xzk$ and C is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9,\ z > 0$.

(2) Find area enclosed by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by Green's theorem.

(3) Verify Gauss Divergence theorem for $\overline{F} = 4xzi - y^2 j + yzk$ and S is the surface of the cube bounded by the planes $x=0,\ x=1,\ y=0,\ y=1,\ z=0, z=1$.

4 (a) Attempt any **two** : **6**

(1) Solve : $(mz-ny)p+(nx-lz)q = ly-mx$

(2) Solve : $xp+yq=3z$

(3) Solve : $\frac{y^2 z}{x}p+xzq=y^2$.

(b) Attempt any **two** : **14**

(1) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, using the method of separation of variable.

(2) Solve the boundary value problem :

$u_{xx} + u_{yy} = 0$, where

$u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$,

for $0 \le x \le l$.
$0 \le y \le a$

(3) A tightly stretched string with fixed end points $x = 0, x = l$ is initially at rest in its equillbrium position. If it is set vibrating giving each point a velocity $\lambda x(l - x)$, find the displacement $u(x, t)$.

5 Attempt any **three** : **15**

(1) Calculate the co-efficient of correlation for the following ages of husbands and wives :

Husband's age x :	23	27	28	28	29	30	31	33	35	36
Wives age y :	18	20	22	27	21	29	27	29	28	29

(2) The following are measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine :

Air Velocity $x\ cm/\text{sec}$	20	60	100	140	180	220	260	300	340	380
Evaporation coefficient $y\ mm^2/\text{sec}$	0.18	0.37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

fit straight line to these data by the method of least squares, and use it to estimate the evaporation coefficient of a droplet when the air velocity is 190 cm/s.

(3) Given data below, find the regression line,

x	20	11	15	10	17	19
y	5	15	14	17	8	9

(4) Obtain the rank correlation coefficient for the following data :

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

6 Attempt any **three** : **15**

(1) 10% of the bolts produced by a certain machine turn out to be defective. Find the probability that in a sample of 10 tools selected at random exactly two will be defective using,

(i) Binomial distribution

(ii) Poisson distribution

And comment on the result.

(2) Assume that the probability of an individual coalminer being killed in a mine accident during a year is $\frac{1}{2400}$.

Use Poisson's distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

(3) A sample of 100 dry battery cells tested to find the length of life produced the following results :

$\bar{x} = 12$ hours, $\sigma = 3$ hours.

Assuming the data to be normally distributed, what percentage on battery cells are expected to have life,

(i) more than 15 hours

(ii) less than 6 hours

(iii) between 10 and 14 hours.

(4) A random variable X has the following probability function :

$X = x$	0	1	2	3	4	5	6	7
$p(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find the value of k

(ii) Find $p(x \leq 5)$ and $p(x > 5)$

(iii) Find $p(0 < x < 6)$